

On the Spectra of Turbulent Velocity Field in a Stably Stratified Flow

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Z. Naturforsch. **46a**, 462–468 (1991); received April 21, 1988

This paper describes a method to solve the spectral equation for the balance of turbulent kinetic energy in a stably stratified turbulent shear flow. The cospectra of vertical momentum and heat flux are modelled with the aid of a basic eddy-viscosity (or turbulent exchange coefficient) function. For the term representing the inertial transfer of turbulent kinetic energy, Pao's [Phys. Fluids **8** (1965)] form is assumed. Analytical expressions for the three-dimensional kinetic energy spectrum as well as the cospectra of momentum and heat flux are obtained over the range of wave numbers $k \geq k_b$, which includes the inertial subrange $k_b \ll k \ll k_s$ and the viscous subrange $k > k_s$ (k_b and k_s are the buoyancy and Kolmogorov wavenumbers, respectively). The two one-dimensional spectra, e.g., the kinetic energy spectra of the horizontal and vertical components of turbulence are derived from the three-dimensional kinetic energy spectrum. These one-dimensional spectra are compared with the measured data of Gargett et al. [J. Fluid Mech. **144** (1984)] for the case I ($=k_s/k_b$) = 630. Finally, we compute the basic eddy-viscosity function and discuss its behaviour.

1. Introduction

Turbulence produced in the atmospheric or oceanic shear zones is often subjected to vertical temperature stratification. If z is the altitude, then the mean temperature gradient $d\bar{T}/dz > 0$ corresponds to stable stratification and $d\bar{T}/dz < 0$ corresponds to unstable stratification. In case of stable stratification the buoyancy forces transform a part of the energy of turbulent fluctuations into potential energy of the flow. In case of unstable stratification, the buoyancy forces transform a part of the potential energy into kinetic energy, i.e. they generate convection.

In the present paper we shall restrict ourselves to the case of stable stratification and further assume that the turbulence is statistically stationary and horizontally homogeneous with a vertical gradient of horizontal mean velocity $d\bar{u}/dz$. In this case the rate of production of turbulent energy due to the work of Reynolds stress on the mean shear is represented by $-\overline{u'w'} \frac{d\bar{u}}{dz}$, where u' and w' are the turbulent velocity components in the horizontal and vertical directions, respectively. The general picture of this kind of turbulence in spectral terms may be described [1] as follows:

The energy is fed into the energy containing range of the spectrum of turbulence by the mean shear and extracted by the buoyancy forces of stable stratification. The net energy is then transferred through a cascade process to larger and larger wave numbers until ultimately the rate of viscous dissipation becomes significant.

The length scale of the energy containing eddies is of the order of l , the length scale of the mean flow as a whole. As discussed by Phillips [1], for motions of the eddies associated with the wave numbers $k \gg l^{-1}$, turbulence can be regarded as locally homogeneous.

For $k \gg l^{-1}$, the spectral equation for the kinetic energy balance [2, 3] in a thermally stratified turbulent shear flow can be written as

$$\varepsilon = F(k) - \frac{d\bar{u}}{dz} \int_k^\infty \tau(k') dk' + \beta \int_k^\infty H(k') dk' + 2\nu \int_0^k k'^2 E(k') dk'. \quad (1)$$

The first term on the right hand side of (1) represents the inertial transfer of turbulent kinetic energy, the second term describes the production of turbulent kinetic energy by the mean velocity shear, the third term describes the contribution of the buoyancy forces towards the turbulent kinetic energy, while the last term describes the dissipation of turbulent kinetic energy under the influence of viscosity; ε is the total dissipa-

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tion of turbulent kinetic energy; $\beta (=g/\bar{T})$ is the buoyancy parameter, where g is the acceleration due to gravity; ν is the coefficient of kinematic viscosity; $E(k)$ is the three-dimensional turbulent kinetic energy spectrum; $\tau(k) [=E_{uw}(k)]$ is the spectrum of vertical momentum flux; $H(k) [=E_{wT}(k)]$ is the spectrum of vertical heat flux. We note the relations

$$\overline{u'w'} = \int_0^\infty E_{uw}(k) dk; \quad \overline{w'T'} = \int_0^\infty E_{wT}(k) dk.$$

The influence of the buoyancy forces is not necessarily restricted to wave numbers of order l^{-1} . A wave number k_b , called the buoyancy wave number is defined as $k_b = (N^3/\varepsilon)^{1/2}$, where N is the Brunt-Väisälä frequency defined by $N = (\beta d\bar{T}/dz)^{1/2}$. For $k \gg k_b$, buoyancy is not important. Lumley [4] has argued that buoyancy acts as a sink in a wave number subrange $l^{-1} \ll k \ll k_b$. Bolgiano [5] and Lumley [4] have shown that the velocity spectra are steeper than $k^{-5/3}$ in the buoyancy subrange. Ozmidov [6] has described the length scale $L_0 (=k_b^{-1})$ as the largest possible turbulent scale allowed by buoyancy forces (within a scaling factor of order 1). This dissipation of turbulent kinetic energy under the influence of viscosity becomes important at the wave number $k_s = (\varepsilon/\nu^3)^{1/4}$, called the Kolmogorov wave number. For sufficiently high Reynolds number, based on Kolmogorov's concept of local isotropy, a range of wave numbers $k_b \ll k \ll k_s$ is expected in which the spectral energy flux is substantially constant. This subrange is called the inertial subrange. In this subrange the energy spectrum $E(k)$ falls off as $k^{-5/3}$. At very high wave numbers $k > k_s$, dissipation of turbulent kinetic energy due to viscosity dominates. This subrange is termed the viscous subrange. The discussions made above suggest that turbulence predominates in the entire range of wave numbers $k \geq k_b$, which includes the inertial and viscous subranges. Here, we attempt to provide a unified description of the turbulent kinetic energy spectrum $E(k)$ for the range of wave numbers $k \geq k_b$ as solution of the energy balance equation (1). We must consider several parameters for this purpose.

For estimation of the background stratification, the flux Richardson number R_f , which is defined as the ratio of the turbulent kinetic energy loss to potential energy to the turbulent kinetic energy gained from the shear of the basic flow, is considered. $R_f = \alpha R_i$, where α is the ratio of the coefficients of turbulent mixing for heat and momentum, and R_i is the gradient Richardson number.

In our analysis α , R_i , and the Brunt Väisälä frequency N will enter as parameters. We restrict ourselves also to the case in which ε , the rate of total dissipation of turbulent kinetic energy by molecular viscosity is not negligible in comparison with the rate of damping of turbulent energy by the stable temperature stratification. We treat ε as a parameter. We consider ν also as parameter, since the way in which the dissipation of turbulent kinetic energy is distributed among the high wave numbers depends on it.

For closure of (1), we model the first three terms of its right hand side. In Sect. 2, we describe the modelling of such terms and obtain analytical expressions for the energy spectrum $E(k)$ and the cospectra $E_{uw}(k)$ and $E_{wT}(k)$ in their nondimensional forms.

In Sect. 3, we derive two one-dimensional spectra, i.e. the energy spectra corresponding to the horizontal and vertical components of turbulent velocity, also in nondimensional forms. In Sect. 4, we compute the basic eddy-viscosity function as suggested in the present analysis.

2. Solutions for the Three-dimensional Kinetic Energy Spectrum, Momentum Flux Spectrum and Heat Flux Spectrum

For the term $F(k)$, which represents the inertial transfer of turbulent kinetic energy through the hierarchy of eddies, we use Pao's [7] form

$$F(k) = P^{-1} \varepsilon^{1/3} k^{5/3} E(k), \quad (2)$$

where P is a dimensionless constant.

The form of $F(k)$ as given by (2) may be interpreted as

$$F(k) = \nu_T (\text{vorticity})^2. \quad (3)$$

If the vorticity in (3) is approximated by $(\varepsilon k^2)^{1/3}$ (cf. Panchev [8]), then a dimensionally correct expression follows in view of (2), as

$$\nu_T \sim k^{1/3} \varepsilon^{-1/3} E(k). \quad (4)$$

As Pao's hypothesis is a local one [8], an explanation of this formulation of ν_T may be advanced in the following way:

It is generally accepted that the effect of turbulence can be reduced exclusively to an increase in friction in the fluid. The turbulent kinematic viscosity ought to be expressible, in Boussinesq's sense, as the product of a characteristic length and a characteristic velocity. In

analogy to this idea, we may write [9] $v_T \sim l_k v_k$, where l_k and v_k are the characteristic length and the characteristic velocity, respectively. Now, if we assume that v_k is determined by the approximate amount of kinetic energy of an eddy centered around a wave number k (cf. Claussen [10]), i.e. $v_k \sim [k E(k)]^{1/2}$, then it follows from (4) that $l_k \sim [k^{-1/3} \varepsilon^{-2/3} E(k)]^{1/2}$. This expression for l_k is apparent from the conjecture of Lin [11] that the characteristic time scale t_k of the energy cascade is given by $t_k \sim \varepsilon^{-1/3} k^{-2/3}$. In effect, we put

$$\begin{aligned} v_T &= P^{-1} [k E(k)]^{1/2} [k^{-1/3} \varepsilon^{-2/3} E(k)]^{1/2} \\ &= P^{-1} k^{1/3} \varepsilon^{-1/3} E(k). \end{aligned} \quad (5)$$

We employ this expression of v_T in our subsequent analysis. Considering the interaction between the mean and turbulent motions to be weak [2, 12], the second and third terms on the right hand side of (1) can be modelled as

$$\frac{d\bar{u}}{dz} \int_k^\infty \tau(k') dk' = -v_T \left(\frac{d\bar{u}}{dz} \right)^2, \quad (6)$$

$$\beta \int_k^\infty H(k') dk' = -v_T^* \left(\beta \frac{d\bar{T}}{dz} \right), \quad (7)$$

where $v_T^* = \alpha v_T$ (cf. Vinnichenko *et al.* [2], Lumley and Panofsky [13]), α being the ratio of the coefficients of turbulent mixing for heat and momentum. Note that $\beta \frac{d\bar{T}}{dz}$ has the dimension of the square of the vorticity.

Substituting (2), (6), and (7) in (1), we obtain

$$\begin{aligned} \varepsilon &= 2\nu \int_0^k k'^2 E(k') dk' + P^{-1} \varepsilon^{1/3} k^{5/3} E(k) \\ &\quad + b P^{-1} \varepsilon^{-1/3} k^{1/3} E(k), \end{aligned} \quad (8)$$

$$\text{where } b = \left(\frac{d\bar{u}}{dz} \right)^2 - \alpha \beta \frac{d\bar{T}}{dz}.$$

Note that 'b' can be written as

$$b = N^2 \left(\frac{1 - \alpha R_i}{R_i} \right).$$

We may rewrite (8) in the form

$$\begin{aligned} \varepsilon &= 2\nu \int_0^k k'^2 E(k') dk' + P^{-1} \varepsilon^{1/3} [k^{5/3} E(k)] \\ &\quad + b P^{-1} \varepsilon^{-5/3} k^{-4/3} [k^{5/3} E(k)], \end{aligned} \quad (9)$$

Differentiating (9) with respect to k and simplifying, we obtain

$$\frac{d[k^{5/3} E(k)]}{k^{5/3} E(k)} = \frac{\frac{4}{3} b \varepsilon^{-1/3} k^{-7/3} - 2\nu P k^{1/3}}{\varepsilon^{1/3} + b \varepsilon^{-1/3} k^{-4/3}}. \quad (10)$$

The solution of (9) is given by

$$k^{5/3} E(k) = Q \exp \int \left(\frac{4}{3} b \varepsilon^{-1/3} k^{-7/3} - 2\nu P k^{1/3} \right) \varepsilon^{-1/3} \cdot (1 + b \varepsilon^{-2/3} k^{-4/3})^{-1} dk, \quad (11)$$

where Q is a constant.

Now

$$\begin{aligned} (1 + b \varepsilon^{-2/3} k^{-4/3})^{-1} &= \left[1 + \frac{1 - \alpha R_i}{R_i} \left(\frac{N}{\varepsilon^{1/3}} \right)^2 k^{-4/3} \right]^{-1} \\ &= \left[1 + \frac{1 - \alpha R_i}{R_i} \left(\frac{k_b}{k} \right)^{4/3} \right]^{-1} \\ &\approx 1 - \frac{1 - \alpha R_i}{R_i} \left(\frac{k_b}{k} \right)^{4/3} \end{aligned} \quad (12)$$

(neglecting higher powers, since we assume $\frac{k_b}{k} \leq 1$ and $\frac{1 - \alpha R_i}{R_i} < 1$).

In view of the relation (12), (11) reduces to

$$k^{5/3} E(k) = Q \exp \int \left(\frac{4}{3} b \varepsilon^{-1/3} k^{-7/3} - 2\nu P k^{1/3} \right) \cdot \left[1 - \frac{1 - \alpha R_i}{R_i} \left(\frac{k_b}{k} \right)^{4/3} \right] \varepsilon^{-1/3} dk. \quad (13)$$

Performing integrations in (13), we obtain

$$\begin{aligned} E(k) &= Q' k^{-5/3} + \frac{2\nu P b}{\varepsilon} \exp[-b \varepsilon^{-2/3} k^{-4/3} \\ &\quad - \frac{3}{2} \nu P \varepsilon^{-1/3} k^{4/3} + \frac{1}{2} b^2 \varepsilon^{-4/3} k^{-8/3}], \end{aligned} \quad (14)$$

where Q' is the new constant. To determine Q' , we assume that at

$$k = k_b, \quad E(k) = \phi_b \left[\left(\frac{\varepsilon^3}{N^5} \right)^{1/2} \right]. \quad (15)$$

The choice of $\phi_b = \left(\frac{\varepsilon^3}{N^5} \right)^{1/2}$ follows from the idea that the appropriate scaling velocity for the largest turbulence eddies is given by

$$u_b = \left(\frac{\varepsilon}{N} \right)^{1/2} \quad \text{and} \quad \phi_b = \frac{u_b^2}{k_b} \quad (\text{cf. Gargett et al. [14]}).$$

Substituting (14) in (13), we find Q' as

$$\begin{aligned} Q' &= \phi_b / [k_b^{-5/3} + 2\nu P b / \varepsilon \\ &\quad \cdot \exp(\frac{1}{2} b^2 \varepsilon^{-4/3} k_b^{-8/3} - b \varepsilon^{-2/3} k_b^{-4/3} \\ &\quad - \frac{3}{2} \nu P \varepsilon^{-1/3} k_b^{4/3})]. \end{aligned} \quad (16)$$

We now scale k and $E(k)$ as

$$\hat{k} = \frac{k}{k_s} \quad \text{and} \quad \hat{E} = \frac{E(k)}{\phi_s},$$

where k_s and ϕ_s are given by $k_s = (\varepsilon/\nu^3)^{1/4}$ and $\phi_s = (\varepsilon \nu^5)^{1/4}$, respectively.

Introducing \hat{k} and \hat{E} in (14) with Q' given by (16), we obtain after some calculation

$$\begin{aligned} \hat{E}(\hat{k}) = & \left(\frac{\varepsilon}{\nu N^2} \right)^{+5/4} (I \hat{k})^{-5/3 + 2PA} \\ & \cdot \exp \left[\frac{A^2}{2} (\hat{k}^{-8/3} - I^{8/3}) + A(I^{4/3} - \hat{k}^{-4/3}) \right. \\ & \left. + \frac{3}{2} P(I^{-4/3} - \hat{k}^{4/3}) \right] \end{aligned} \quad (17)$$

where $I = \frac{k_s}{k_b}$ and $A = \frac{\nu}{\varepsilon} \frac{1 - \alpha R_i}{R_i} N^2$.

Clearly, to compute $\hat{E}(\hat{k})$ from (16) we must know the values of the parameters α , R_i , N , ε , ν , and I . Gargett *et al.* [14] have measured all the parameters except α and R_i in the Knight-Inlet fjord. To demonstrate the result (17), we consider $N = 2.1 \times 10^{-2}$ (rad s⁻¹), $\varepsilon = 3.6 \times 10^{-6}$ (m² s⁻³), $\nu = 1.48 \times 10^{-6}$ (m² s⁻¹) from Table 2 of [14], which corresponds to their measurements of ϕ_{11} and ϕ_{33} for $I = 630$ [class 2 records $600 < I < 900$] and accept some probable values of α and R_i . Bowden [15], in his study of the density current flow in an estuary, has observed that R_i varies from 0.5 to 1. We choose $R_i = 0.8$ and obtain the value of α from the relation.

$\alpha = (1 + 10 R_i)^{1/2} / (1 + 3.3 R_i)^{3/2}$ (cf. Bowden [16]) as 0.432. We shall discuss about the choice $R_i = 0.8$ in the next section wherein we shall compare the one dimensional spectra, to be derived from (17) with the measured data of Gargett *et al.* [14]. The plots of $\log \hat{E}$ vs. $\log \hat{k}$ for the entire range of wave numbers $\hat{k} \geq k_b/k_s$ are shown in Figure 1. At low wave numbers, the modulus of the slope of the spectral density curve does not exceed $\frac{5}{3}$. At some intermediate wave numbers (inertial subrange) \hat{E} exhibits approximately a $\hat{k}^{-5/3}$ fall off (Figure 1). At high wave numbers \hat{E} shows approximately a \hat{k}^{-7} behavior (Fig. 1), after which (at more higher wave numbers) it decays at a rate faster than \hat{k}^{-7} .

We now derive from (6) the spectrum of momentum flux in its nondimensional form, i.e. $\hat{E}_{uw}(\hat{k})$ $\left[= E_{uw}(k) \left/ \left(\frac{d\bar{u}}{dz} \nu^{7/4} \varepsilon^{-1/4} \right) \right. \right]$, utilizing the relations (5)

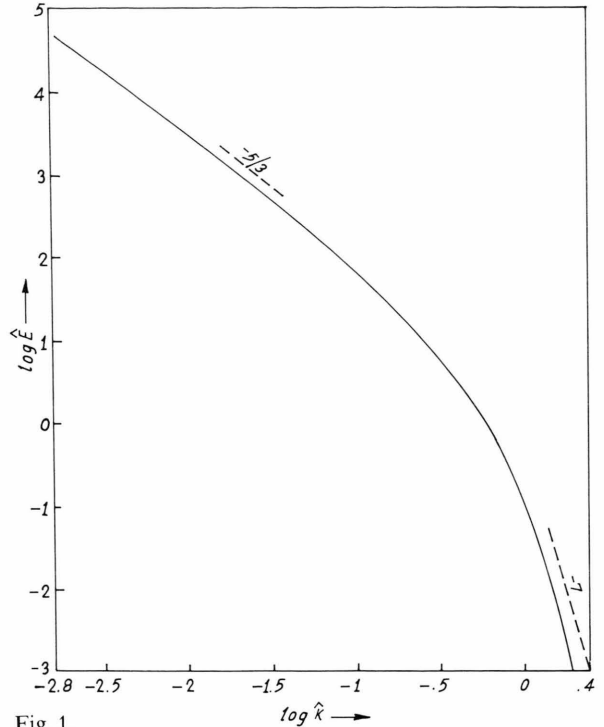


Fig. 1.

and (14)–(17), as

$$\begin{aligned} \hat{E}_{uw}(\hat{k}) = & \frac{1}{P} \left[\left(-\frac{4}{3} + 2PA \right) \hat{k}^{-2/3} + \frac{4}{3} A \hat{k}^{-2} \right. \\ & \left. - \frac{4}{3} A^2 \hat{k}^{-10/3} - 2P \hat{k}^{2/3} \right] \hat{E}(\hat{k}). \end{aligned} \quad (18)$$

The nondimensionalized spectrum of heat flux, i.e. $\hat{E}_{wT}(\hat{k}) \left[= E_{wT}(k) \left/ \left(\frac{dT}{dz} \varepsilon^{-1/4} \nu^{7/4} \right) \right. \right]$, is calculated in the same manner from (7), using (5) and (14)–(17), as

$$\begin{aligned} \hat{E}_{wT}(\hat{k}) = & \frac{\alpha}{P} \left[\left(-\frac{4}{3} + 2PA \right) \hat{k}^{-2/3} + \frac{4}{3} A \hat{k}^{-2} \right. \\ & \left. - \frac{4}{3} A^2 \hat{k}^{-10/3} - 2P \hat{k}^{2/3} \right] \hat{E}(\hat{k}). \end{aligned} \quad (19)$$

We plot $\log[-\hat{E}_{uw}(\hat{k})]$ vs. $\log \hat{k}$ and $\log[-\hat{E}_{wT}(\hat{k})]$ vs. $\log \hat{k}$ (Fig. 2) employing the same values of α , P , N , ε , ν , and R_i as considered in case of computing the three-dimensional kinetic energy spectrum $\hat{E}(\hat{k})$. Clearly, these two cospectra exhibit a reasonable behaviour [17], i.e. $a - \frac{7}{3}$ power law in the inertial subrange.

The determination of the spectrum of temperature variance is also of considerable interest, as stratification reflects the distribution of temperature in the flow. This can be accomplished by solving the equation for the temperature variance balance. We shall consider this problem in another paper.

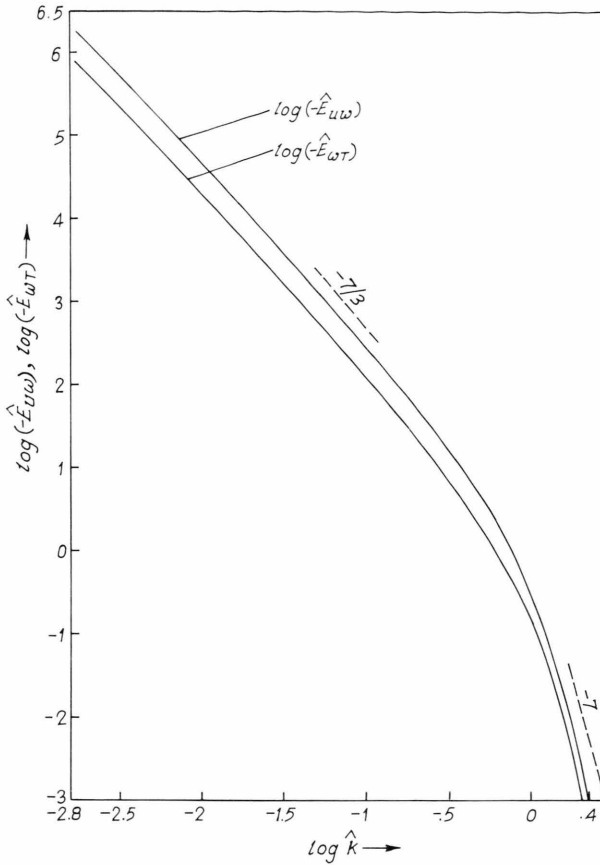


Fig. 2.

3. Kinetic Energy Spectra of Horizontal and Vertical Components of Turbulence

As measurements are often made along straight lines, it is important to deduce one-dimensional spectra theoretically. For this purpose we use the relations between three-dimensional and one-dimensional kinetic energy spectra [18, 19]. These relations between three-dimensional and one-dimensional kinetic energy spectra in nondimensional form are

$$\hat{\phi}_{11} (= \varphi_{11}/\varphi_s) = \int_{\hat{k}}^{\infty} \left(1 - \frac{\hat{k}^2}{\hat{\eta}^2}\right) \frac{\hat{E}(\hat{\eta})}{\hat{\eta}} d\hat{\eta} \quad (20)$$

and

$$\begin{aligned} \hat{\phi}_{33} \left(= \frac{\varphi_{33}}{\varphi_s} \right) &= \hat{\phi}_{22} \left(= \frac{\varphi_{22}}{\varphi_s} \right) \\ &= \frac{1}{2} \int_{\hat{k}}^{\infty} \left(1 + \frac{\hat{k}^2}{\hat{\eta}^2}\right) \frac{\hat{E}(\hat{\eta})}{\hat{\eta}} d\hat{\eta}, \end{aligned} \quad (21)$$

where $\hat{\phi}_{11}$, $\hat{\phi}_{22}$, and $\hat{\phi}_{33}$ are the non-dimensional u -component, v -component and w -component energy spectra, respectively.

Putting $\sigma = \frac{\hat{k}}{\hat{\eta}}$ and using the conditions $\sigma \rightarrow 1$ as $\hat{\eta} \rightarrow \hat{k}$ and $\sigma \rightarrow 0$ as $\hat{\eta} \rightarrow \infty$, we transform (20) and (21) into

$$\hat{\phi}_{11} = \int_0^1 \frac{1-\sigma^2}{\sigma} \hat{E}\left(\frac{\hat{k}}{\sigma}\right) d\sigma \quad (22)$$

and

$$\hat{\phi}_{33} = \frac{1}{2} \int_0^1 \frac{1+\sigma^2}{\sigma} \hat{E}\left(\frac{\hat{k}}{\sigma}\right) d\sigma. \quad (23)$$

In view of (17), the relations (22) and (23) reduce, respectively, to the forms

$$\begin{aligned} \hat{\phi}_{11} &= \int_0^1 \left[\frac{1-\sigma^2}{\sigma} \left(\frac{\varepsilon}{v N^2} \right)^{5/4} (I \hat{k})^{-5/3+2PA} \sigma^{5/3-2PA} \right. \\ &\quad \cdot \exp \left\{ \frac{A^2}{2} (\hat{k}^{-8/3} \sigma^{8/3} - I^{8/3}) + A(I^{4/3} - \hat{k}^{-4/3} \sigma^{4/3}) \right. \\ &\quad \left. \left. + \frac{3}{2} P(I^{-4/3} - \hat{k}^{4/3} \sigma^{-4/3}) \right\} \right] d\sigma \end{aligned} \quad (24)$$

and

$$\begin{aligned} \hat{\phi}_{33} &= \frac{1}{2} \int_0^1 \left[\frac{1+\sigma^2}{\sigma} \left(\frac{\varepsilon}{v N^2} \right)^{5/4} (I \hat{k})^{-5/3+2PA} \sigma^{5/3-2PA} \right. \\ &\quad \cdot \exp \left\{ \frac{A^2}{2} (\hat{k}^{-8/3} \sigma^{8/3} - I^{8/3}) + A(I^{4/3} - \hat{k}^{-4/3} \sigma^{4/3}) \right. \\ &\quad \left. \left. + \frac{3}{2} P(I^{-4/3} - \hat{k}^{4/3} \sigma^{-4/3}) \right\} \right] d\sigma. \end{aligned} \quad (25)$$

We now calculate $\hat{\phi}_{11}$ and $\hat{\phi}_{33}$, respectively, from (24) and (25) for the same set of values of the parameters as considered in Sect. 2 for demonstrating the three dimensional energy spectrum \hat{E} . The plots of $\log \hat{\phi}_{11}$ vs. $\log \hat{k}$ and $\log \hat{\phi}_{33}$ vs. $\log \hat{k}$ are shown in Figs. 3 and 4, respectively. For comparisons, we have also plotted $\log \hat{\phi}_{11}$ and $\log \hat{\phi}_{33}$ for $I = 630$ (with symbol \blacktriangledown) from the experimental data of Gargett *et al.* [14]. Inspection of these figures indicates that the present theoretical calculations of $\log \hat{\phi}_{11}$ and $\log \hat{\phi}_{33}$ agree well with their respective measured values. It is to be mentioned here that the value of R_i i.e. 0.8, was chosen for the theoretical calculations in order to obtain this agreement.

The measurements (class 1 records) of Gargett *et al.* [14] indicate that for $I \gtrsim 3000$, i.e. when k_b lies at least

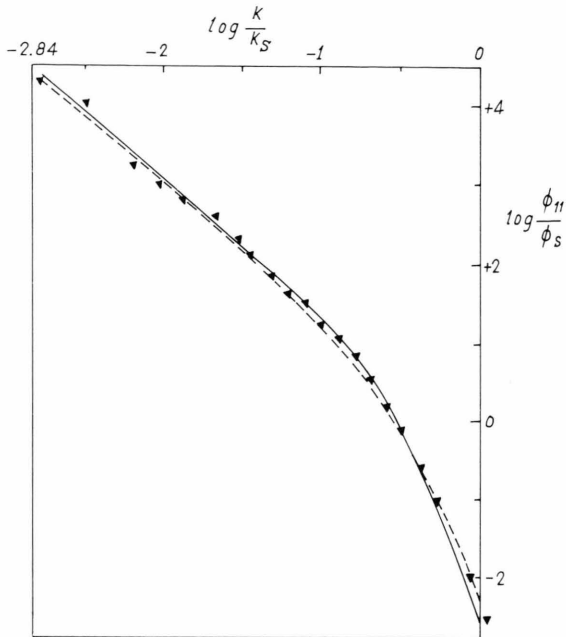


Fig. 3.

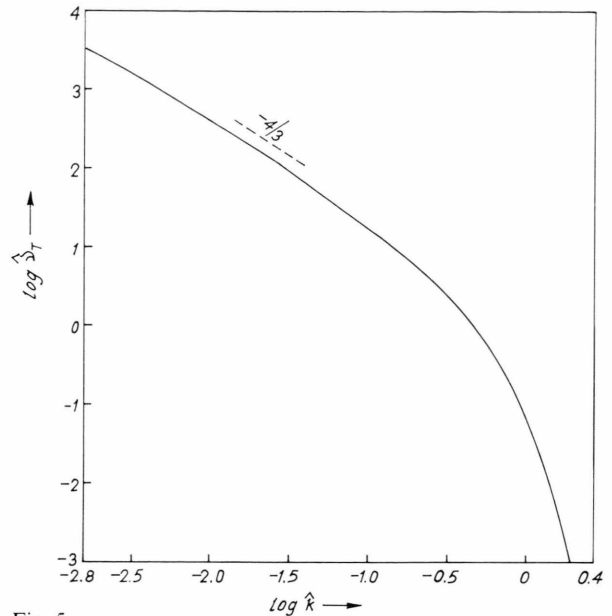


Fig. 5.

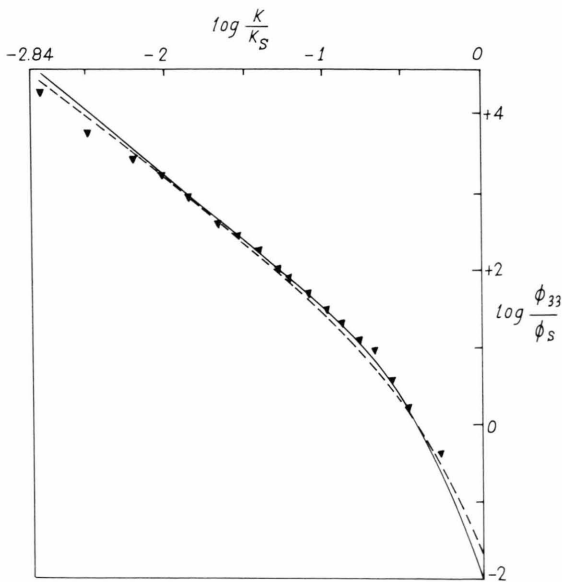


Fig. 4.

three decades in wave number below the wave number k_s by which the velocity gradients are wiped out by molecular diffusion, all the spectral components possess substantial $-\frac{5}{3}$ subranges. Also in their measurements for the range $200 < I < 300$ (class 3 records), the cross-stream spectra exhibit clear -1 subranges and

the axial spectrum maintains an acceptable $-\frac{5}{3}$ subrange (values are beginning to fall slightly below the universal curve). Although the specific stages in turbulent decay can be identified when spectra take on distinctive shapes, the evolution between these stages is continuous. From these, it seems that for the range $600 < I < 900$ (class 2 records) the existence of velocity gradients at least in small amounts may not possibly be excluded. For the present case ($I = 630$), the velocity gradient du/dz as calculated from the relation $R_i = N^2 / (d\bar{u}/dz)^2$ with $R_i = 0.8$ and $N = 2.1 \times 10^{-2}$ (rad s $^{-1}$), is 0.023 (s $^{-1}$). This value of $d\bar{u}/dz$ is considerably small. Gargett *et al.* [14] have provided evidence of developing anisotropy at low wave numbers for the range $600 < I < 900$. Both the present theoretical calculations and the measured data for $I = 630$ show that $\hat{\phi}_{11}$ and $\hat{\phi}_{33}$ possess approximate $\hat{k}^{-5/3}$ behaviour for short subranges only. This may be caused by the insufficiency of the 'room' in wave number space for the development of an inertial cascade when $I = 630$. Comparison between the Figs. 3 and 4 suggests that the anisotropy developed in this case is weak.

4. Computation of the Basic Eddy-Viscosity Function

In order to express the basic eddy-viscosity function ν_T in nondimensional form, we introduce \hat{k} and

\hat{E} in (5):

$$\begin{aligned} v_T &= P^{-1} \varepsilon^{-1/3} k^{1/3} E(k) \\ &= P^{-1} \varepsilon^{-1/3} \hat{k}^{1/3} k_s^{1/3} \hat{E} \phi_s \\ &= P^{-1} \varepsilon^{-1/3} [(\varepsilon/v^3)^{1/4}]^{1/3} (\varepsilon v^5)^{1/4} \hat{k}^{1/3} \hat{E} \\ &= P^{-1} v \hat{k}^{1/3} \hat{E}. \end{aligned}$$

Now, scaling the basic eddy-viscosity function v_T as $\hat{v}_T = v_T/v$, we obtain

$$\hat{v}_T = P^{-1} \hat{k}^{1/3} \hat{E}(\hat{K}). \quad (26)$$

Taking (17) into account, we compute \hat{v}_T from (26) and plot $\log \hat{v}_T$ vs. $\log \hat{k}$ (Figure 5).

It is clear from Fig. 5 that in the region of wavenumbers where the kinetic energy spectrum \hat{E} exhibits $\hat{k}^{-5/3}$ dependence, the spectral eddy-viscosity \hat{v}_T has a shape proportional to $\hat{k}^{-4/3}$. At low wavenumbers

where, the plateau of \hat{v}_T is influenced by shear and stratification, the modulus of the slope of the \hat{v}_T -curve does not exceed 4/3. At higher wavenumbers \hat{v}_T falls off rapidly with the decrease of \hat{E} . Thus the behaviour of \hat{v}_T as determined here for the treated range of wavenumbers is consistent with the idea that spectral eddy-viscosity must depict the properties of the turbulent flow concerned.

Acknowledgement

We thank Professor A. Ghosh (Calcutta), Professor S. Khamrui (Jadavpur, Calcutta) and Professor J. Das (Calcutta) for stimulating discussions on this problem. We express our gratitude to the referee for his valuable suggestions.

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